



Combinatorics B

1. How many ways can you arrange 3 Alice's, 1 Bob, 3 Chad's, and 1 David in a line if the Alice's are all indistinguishable, the Chad's are all indistinguishable, and Bob and David want to be adjacent to each other? (In other words, how many ways can you arrange 3 A's, 1 B, 3 C's, and 1 D in a row where the B and D are adjacent?)
2. Suppose Alan, Michael, Kevin, Igor, and Big Rahul are in a running race. It is given that exactly one pair of people tie (for example, two people both get second place), so that no other pair of people end in the same position. Each competitor has equal skill; this means that each outcome of the race, given that exactly two people tie, is equally likely. The probability that Big Rahul gets first place (either by himself or he ties for first) can be expressed in the form m/n , where m, n are relatively prime, positive integers. Compute $m + n$.
3. Prinstan Trollner and Dukejukem are competing at the game show WASS. Both players spin a wheel which chooses an integer from 1 to 50 uniformly at random, and this number becomes their score. Dukejukem then flips a weighted coin that lands heads with probability $3/5$. If he flips heads, he adds 1 to his score. A player wins the game if their score is higher than the other player's score. The probability Dukejukem defeats the Trollner to win WASS equals m/n where m, n are coprime positive integers. Compute $m + n$.
4. Keith has 10 coins labeled 1 through 10, where the i th coin has weight 2^i . The coins are all fair, so the probability of flipping heads on any of the coins is $\frac{1}{2}$. After flipping all of the coins, Keith takes all of the coins which land heads and measures their total weight, W . If the probability that $137 \leq W \leq 1061$ is m/n for coprime positive integers m, n , determine $m + n$.
5. Marko lives on the origin of the Cartesian plane. Every second, Marko moves 1 unit up with probability $2/9$, 1 unit right with probability $2/9$, 1 unit up and 1 unit right with probability $4/9$, and he doesn't move with probability $1/9$. After 2019 seconds, Marko ends up on the point (A, B) . What is the expected value of $A \cdot B$?
6. Kelvin and Quinn are collecting trading cards; there are 6 distinct cards that could appear in a pack. Each pack contains exactly one card, and each card is equally likely. Kelvin buys packs until he has at least one copy of every card, then he stops buying packs. If Quinn is missing exactly one card, the probability that Kelvin has at least two copies of the card Quinn is missing is expressible as m/n for coprime positive integers m, n . Determine $m + n$.
7. A candy store has 100 pieces of candy to give away. When you get to the store, there are five people in front of you, numbered from 1 to 5. The i th person in line considers the set of positive integers congruent to i modulo 5 which are at most the number of pieces of candy remaining. If this set is empty, then they take no candy. Otherwise they pick an element of this set and take that many pieces of candy. For example, the first person in line will pick an integer from the set $\{1, 6, \dots, 96\}$ and take that many pieces of candy. How many ways can the first five people take their share of candy so that after they are done there are at least 35 pieces of candy remaining?
8. The Nationwide Basketball Society (NBS) has 8001 teams, numbered 2000 through 10000. For each n , team n has $n + 1$ players, and in a sheer coincidence, this year each player attempted n shots and on team n , exactly one player made 0 shots, one player made 1 shot, \dots , one player made n shots. A player's *field goal percentage* is defined as the percentage of shots that the player made, rounded to the nearest tenth of a percent. (For instance, 32.45% rounds to 32.5%.) A player in the NBS is randomly selected among those whose field goal percentage is 66.6%. If this player plays for team k , what is the probability that $k \geq 6000$?