Geometry A

1. Let γ_1 and γ_2 be circles centered at O and P respectively, and externally tangent to each other at point Q. Draw point D on γ_1 and point E on γ_2 such that line DE is tangent to both circles. If the length $OQ = 1$ and the area of the quadrilateral $ODEP$ is 520, then what is the value of length PQ?

2. Hexagon $ABCDEF$ has an inscribed circle Ω that is tangent to each of its sides. If $AB = 12$, $\angle FAB = 120°$, and $\angle ABC = 150°$, and if the radius of Ω can be written as $m + \sqrt{n}$ for positive integers m, n, find $m + n$.

3. Let $ABCD$ be a cyclic quadrilateral with circumcenter O and radius 10. Let sides AB, BC, CD, and DA have midpoints M, N, P, and Q, respectively. If $MP = NQ$ and $OM + OP = 16$, then what is the area of triangle $\triangle OAB$?

4. Let C be a circle centered at point O, and let P be a point in the interior of C. Let Q be a point on the circumference of C such that $PQ \perp OP$, and let D be the circle with diameter PQ. Consider a circle tangent to C whose circumference passes through point P. Let the curve Γ be the locus of the centers of all such circles. If the area enclosed by Γ is $1/100$ the area of C, then what is the ratio of the area of C to the area of D?

5. Triangle ABC is so that $AB = 15, BC = 22$, and $AC = 20$. Let D, E, F lie on BC, AC, and AB, respectively, so AD, BE, CF all contain a point K. Let L be the second intersection of the circumcircles of BFK and CEK. Suppose that $\frac{AK}{KD} = \frac{11}{7}$, and $BD = 6$. If $KL^2 = \frac{a}{b}$, where a, b are relatively prime integers, find $a + b$.

6. Triangle ABC has side lengths 13, 14, and 15. Let E be the ellipse that encloses the smallest area which passes through A, B, and C. The area of E is of the form $\frac{a\sqrt{b}\pi}{c}$, where a and c are coprime and b has no square factors. Find $a + b + c$.

7. Let ABC be a triangle with sides $AB = 34, BC = 15, AC = 35$ and let Γ be the circle of smallest possible radius passing through A tangent to BC. Let the second intersections of Γ and sides AB, AC be the points X, Y. Let the ray XY intersect the circumcircle of the triangle ABC at Z. If $AZ = \frac{p}{q}$ for relatively prime integers p and q, find $p + q$.

8. $A_1A_2A_3A_4$ is a cyclic quadrilateral inscribed in circle Ω, with side lengths $A_1A_2 = 28$, $A_2A_3 = 12\sqrt{3}$, $A_3A_4 = 28\sqrt{3}$, and $A_4A_1 = 8$. Let X be the intersection of A_1A_3, A_2A_4. Now, for $i = 1, 2, 3, 4$, let ω_i be the circle tangent to segments $A_iX, A_{i+1}X$, and Ω, where we take indices cyclically $\pmod 4$. Furthermore, for each i, say ω_i is tangent to A_1A_3 at X_i, A_2A_4 at Y_i, and Ω at T_i. Let P_1 be the intersection of T_1X_1 and T_2X_2, and P_3 the intersection of T_3X_3 and T_4X_4. Let P_2 be the intersection of T_2Y_2 and T_3Y_3, and P_4 the intersection of T_1Y_1 and T_4Y_4. Find the area of quadrilateral $P_1P_2P_3P_4$.