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Geometry B

1. You are walking along a road of constant width with sidewalks on each side. You can only walk on the sidewalks or cross the road perpendicular to the sidewalk. Coming up on a turn, you realize that you are on the "outside" of the turn; i.e., you are taking the longer way around the turn. The turn is a circular arc. Assuming that your destination is on the same side of the road as you are currently, let θ be the smallest turn angle, in radians, that would justify crossing the road and then crossing back after the turn to take the shorter total path to your destination. What is $\lfloor 100 \times \theta \rfloor$?

2. Seven students in Princeton Juggling Club are searching for a room to meet in. However, they must stay at least 6 feet apart from each other, and due to midterms, the only open rooms they can find are circular. In feet, what is the smallest diameter of any circle which can contain seven points, all of which are at least 6 feet apart from each other?

3. Let γ_1 and γ_2 be circles centered at O and P respectively, and externally tangent to each other at point Q. Draw point D on γ_1 and point E on γ_2 such that line DE is tangent to both circles. If the length $OQ = 1$ and the area of the quadrilateral $ODEP$ is 520, then what is the value of length PQ?

4. Hexagon $ABCDEF$ has an inscribed circle Ω that is tangent to each of its sides. If $AB = 12$, $\angle FAB = 120°$, and $\angle ABC = 150°$, and if the radius of Ω can be written as $m + \sqrt{n}$ for positive integers m, n, find $m + n$.

5. Let $ABCD$ be a cyclic quadrilateral with circumcenter O and radius 10. Let sides AB, BC, CD, and DA have midpoints M, N, P, and Q, respectively. If $MP = NQ$ and $OM + OP = 16$, then what is the area of triangle $\triangle OAB$?

6. Let C be a circle centered at point O, and let P be a point in the interior of C. Let Q be a point on the circumference of C such that $PQ \perp OP$, and let D be the circle with diameter PQ. Consider a circle tangent to C whose circumference passes through point P. Let the curve Γ be the locus of the centers of all such circles. If the area enclosed by Γ is $1/100$ the area of C, then what is the ratio of the area of C to the area of D?

7. Triangle ABC is so that $AB = 15, BC = 22$, and $AC = 20$. Let D, E, F lie on BC, AC, and AB, respectively, so AD, BE, CF all contain a point K. Let L be the second intersection of the circumcircles of BFK and CEK. Suppose that $\frac{AK}{KD} = \frac{11}{7}$, and $BD = 6$. If $KL^2 = \frac{a}{b}$, where a, b are relatively prime integers, find $a + b$.

8. Triangle ABC has side lengths 13, 14, and 15. Let E be the ellipse that encloses the smallest area which passes through A, B, and C. The area of E is of the form $\frac{a\sqrt{b}\pi}{c}$, where a and c are coprime and b has no square factors. Find $a + b + c$.