Geometry A

1. A circle is inscribed in a regular octagon with area 2024. A second regular octagon is inscribed in the circle, and its area can be expressed as $a + b\sqrt{c}$, where a, b, c are integers and c is square-free. Compute $a + b + c$.

2. Triangle $\triangle ABC$ has $AB = 8$, $AC = 10$, and $AD = \sqrt{33}$, where D is the midpoint of BC. Perpendiculars are drawn from D to meet AB and AC at E and F, respectively. The length of EF can be expressed as $\frac{a\sqrt{b}}{c}$, where a, c are relatively prime and b is square-free. Compute $a + b + c$.

3. Consider a circle centered at O. Parallel chords AB of length 8 and CD of length 10 are of distance 2 apart such that $AC < AD$. We can write $\tan \angle BOD = \frac{a}{b}$, where a, b are positive integers such that $\gcd(a, b) = 1$. Compute $a + b$.

4. Let $BCDE$ be a trapezoid with $BE \parallel CD$, $BE = 20, BC = 2\sqrt{34}, CD = 8, DE = 2\sqrt{10}$. Draw a line through E parallel to BD and a line through B perpendicular to BE, and let A be the intersection of these two lines. Let M be the intersection of diagonals BD and CE, and let X be the intersection of AM and BE. If BX can be written as $\frac{a}{b}$, where a, b are relatively prime positive integers, find $a + b$.

5. A pentagon has vertices labelled A, B, C, D, E in that order counterclockwise, such that AB, ED are parallel and $\angle EAB = \angle ABD = \angle ACD = \angle CDA$. Furthermore, suppose that $AB = 8, AC = 12, AE = 10$. If the area of triangle CDE can be expressed as $\frac{a\sqrt{b}}{c}$, where a, b, c are integers so that b is square free, and a, c are relatively prime, find $a + b + c$.

6. Three circles, $\omega_1, \omega_2, \omega_3$ are drawn, with ω_3 externally tangent to ω_1 at C and internally tangent to ω_2 at D. Say also that $\omega_1\omega_2$ intersect at points A, B. Suppose the radius of ω_1 is 20, the radius of ω_2 is 15, and the radius of ω_3 is 6. Draw line CD, and suppose it meets AB at point X. If $AB = 24$, then CX can be written in the form $\frac{a\sqrt{b}}{c}$, where a, b, c are positive integers where b is square-free, and a, c are relatively prime. Find $a + b + c$.

7. Let ABC be a triangle with side lengths $AB = 13, AC = 17$, and $BC = 20$. Let E, F be the feet of the altitudes from B onto AC and C onto AB, respectively. Let P be the second intersection of the circumcircles of ABC and AEF. Suppose that AP can be written as $\frac{a\sqrt{b}}{c}$ where a, c are relatively prime and b is square-free. Compute a.

8. Let ABC be an acute triangle with side lengths $AB = 7, BC = 12, AC = 10$, and let ω be its incircle. If ω is touching AB, AC at F, E, respectively, and if EF intersects BC at X, suppose that the ratio in which the angle bisector of $\angle BAC$ divides the segment connecting midpoint of EX and C is $\frac{a}{b}$, where a, b are relatively prime integers. Find $a + b$.