Geometry B

1. Parallelogram $ABCD$ is given such that $\angle ABC$ equals 30°. Let X be the foot of the perpendicular from A onto BC, and Y the foot of the perpendicular from C to AB. If $AX = 20$ and $CY = 22$, find the area of the parallelogram.

2. A right cylinder is given with a height of 20 and a circular base of radius 5. A vertical planar cut is made into this base of radius 5. A vertical planar cut, perpendicular to the base, is made into this cylinder, splitting the cylinder into two pieces. Suppose the area the cut leaves behind on one of the pieces is $100\sqrt{2}$. Then the volume of the larger piece can be written as $a + b\pi$, where a, b are positive integers. Find $a + b$.

3. A circle is inscribed in a regular octagon with area 2024. A second regular octagon is inscribed in the circle, and its area can be expressed as $a + b\sqrt{c}$, where a, b, c are integers and c is square-free. Compute $a + b + c$.

4. Triangle $\triangle ABC$ has $AB = 8$, $AC = 10$, and $AD = \sqrt{33}$, where D is the midpoint of BC. Perpendiculars are drawn from D to meet AB and AC at E and F, respectively. The length of EF can be expressed as $\frac{a\sqrt{b}}{c}$, where a, c are relatively prime positive integers and b is square-free. Compute $a + b + c$.

5. Consider a circle centered at O. Parallel chords AB of length 8 and CD of length 10 are of distance 2 apart such that $AC < AD$. We can write $\tan\angle BOD = \frac{a}{b}$, where a, b are positive integers such that $\gcd(a, b) = 1$. Compute $a + b$.

6. Let $BCDE$ be a trapezoid with $BE \parallel CD$, $BE = 20, BC = 2\sqrt{34}, CD = 8, DE = 2\sqrt{10}$. Draw a line through E parallel to BD and a line through B perpendicular to BE, and let A be the intersection of these two lines. Let M be the intersection of diagonals BD and CE, and let X be the intersection of AM and BE. If BX can be written as $\frac{a}{b}$, where a, b are relatively prime positive integers, find $a + b$.

7. A pentagon has vertices labelled A, B, C, D, E in that order counterclockwise, such that AB, ED are parallel and $\angle EAB = \angle ABD = \angle ACD = \angle CDA$. Furthermore, suppose that $AB = 8, AC = 12, AE = 10$. If the area of triangle CDE can be expressed as $\frac{a\sqrt{b}}{c}$, where a, b, c are integers so that b is square free, and a, c are relatively prime, find $a + b + c$.

8. Three circles, $\omega_1, \omega_2, \omega_3$ are drawn, with ω_3 externally tangent to ω_1 at C and internally tangent to ω_2 at D. Say also that $\omega_1\omega_2$ intersect at points A, B. Suppose the radius of ω_1 is 20, the radius of ω_2 is 15, and the radius of ω_3 is 6. Draw line CD, and suppose it meets AB at point X. If $AB = 24$, then CX can be written in the form $\frac{a\sqrt{b}}{c}$, where a, b, c are positive integers where b is square-free, and a, c are relatively prime. Find $a + b + c$.