Geometry A

1. Frist Campus Center is located 1 mile north and 1 mile west of Fine Hall. The area within 5 miles of Fine Hall that is located north and east of Frist can be expressed in the form $\frac{a}{b}\pi - c$, where a, b, c are positive integers and a and b are relatively prime. Find $a + b + c$.

2. Let $\overline{AD}$ be a diameter of a circle. Let point B be on the circle, point C be on $\overline{AD}$ such that A, B, C form a right triangle with right angle at C. The value of the hypotenuse of the triangle is 4 times the square root of its area. If $\overline{BC}$ has length 30, what is the length of the radius of the circle?

3. Let $\triangle ABC$ satisfy $AB = 17$, $AC = \frac{70}{3}$ and $BC = 19$. Let I be the incenter of $\triangle ABC$ and E be the excenter of $\triangle ABC$ opposite A. (Note: this means that the circle tangent to ray AB beyond B, ray AC beyond C, and side BC is centered at E.) Suppose the circle with diameter IE intersects AB beyond B at D. If $BD = \frac{a}{b}$ where a, b are coprime positive integers, find $a + b$.

4. Triangle ABC has $\angle A = 90^\circ$, $\angle C = 30^\circ$, and $AC = 12$. Let the circumcircle of this triangle be W. Define D to be the point on arc BC not containing A so that $\angle CAD = 60^\circ$. Define points E and F to be the foots of the perpendiculars from D to lines AB and AC, respectively. Let J be the intersection of line EF with W, where J is on the minor arc AC. The line DF intersects W at H other than at D. The area of the triangle FHJ is in the form $\frac{a}{b}(\sqrt{c} - \sqrt{d})$ for positive integers a, b, c, d, where a, b are relatively prime, and the sum of a, b, c, d is minimal. Find $a + b + c + d$.

5. Let $\triangle ABC$ be triangle with side lengths $AB = 9, BC = 10, CA = 11$. Let O be the circumcenter of $\triangle ABC$. Denote $D = AO \cap BC, E = BO \cap CA, F = CO \cap AB$. If $1/AD + 1/BE + 1/FC$ can be written in simplest form as $\frac{a\sqrt{b}}{c}$, find $a + b + c$.

6. Let triangle ABC have $\angle BAC = 45^\circ$ and circumcircle Γ and let M be the intersection of the angle bisector of $\angle BAC$ with Γ. Let Ω be the circle tangent to segments $\overline{AB}$ and $\overline{AC}$ and internally tangent to Γ at point T. Given that $\angle TMA = 45^\circ$ and that $TM = \sqrt{100 - 50\sqrt{2}}$, the length of BC can be written as $a\sqrt{b}$, where b is not divisible by the square of any prime. Find $a + b$.

7. Let $ABCD$ be a parallelogram such that $AB = 35$ and $BC = 28$. Suppose that $BD \perp BC$. Let ℓ_1 be the reflection of AC across the angle bisector of $\angle BAD$, and let ℓ_2 be the line through B perpendicular to CD. ℓ_1 and ℓ_2 intersect at a point P. If PD can be expressed in simplest form as $\frac{m}{n}$, find $m + n$.

8. Let ω be a circle. Let E be on ω and S be outside ω such that line segment SE is tangent to ω. Let R be on ω. Let line SR intersect ω at B other than R, such that R is between S and B. Let I be the intersection of the bisector of $\angle ESR$ with the line tangent to ω at R; let A be the intersection of the bisector of $\angle ESR$ with ER. If the radius of the circumcircle of $\triangle EIA$ is 10, the radius of the circumcircle of $\triangle SAB$ is 14, and $SA = 18$, then IA can be expressed in simplest form as $\frac{m}{n}$. Find $m + n$.