Geometry A

1. Circle Γ is centered at $(0,0)$ in the plane with radius $2022\sqrt{3}$. Circle Ω is centered on the x-axis, passes through the point $A = (6066, 0)$, and intersects Γ orthogonally at the point $P = (x, y)$ with $y > 0$. If the length of the minor arc AP on Ω can be expressed as $\frac{m\pi}{n}$ for relatively prime positive integers m, n, find $m + n$.

   (Two circles intersect *orthogonally* at a point P if the tangent lines at P form a right angle.)

2. An ellipse has foci A and B and has the property that there is some point C on the ellipse such that the area of the circle passing through A, B, and, C is equal to the area of the ellipse. Let e be the largest possible eccentricity of the ellipse. One may write e^2 as $\frac{a+\sqrt{b}}{c}$, where a, b, and c are integers such that a and c are relatively prime, and b is not divisible by the square of any prime. Find $a^2 + b^2 + c^2$.

3. Daeun draws a unit circle centered at the origin and inscribes within it a regular hexagon $ABCDEF$. Then Dylan chooses a point P within the circle of radius 2 centered at the origin. Let M be the maximum possible value of $|PA| \cdot |PB| \cdot |PC| \cdot |PD| \cdot |PE| \cdot |PF|$, and let N be the number of possible points P for which this maximal value is obtained. Find $M + N^2$.

4. Let $\triangle ABC$ be an equilateral triangle. Points D, E, F are drawn on sides AB, BC, and CA respectively such that $[ADF] = [BED] + [CEF]$ and $\triangle ADF \sim \triangle BED \sim \triangle CEF$. The ratio $\frac{[ABC]}{[DEF]}$ can be expressed as $\frac{a+b\sqrt{c}}{d}$, where a, b, c, and d are positive integers such that a and d are relatively prime, and c is not divisible by the square of any prime. Find $a + b + c + d$.

   (Here $[\mathcal{P}]$ denotes the area of polygon $\mathcal{P}$.)

5. Let $\triangle ABC$ be a triangle with $AB = 5$, $BC = 8$, and, $CA = 7$. Let the center of the A-excircle be O, and let the A-excircle touch lines BC, CA, and, AB at points X, Y, and, Z, respectively. Let h_1, h_2, and, h_3 denote the distances from O to lines XY, YZ, and, ZX, respectively. If $h_1^2 + h_2^2 + h_3^2$ can be written as $\frac{m}{n}$ for relatively prime positive integers m, n, find $m + n$.

6. Triangle $\triangle ABC$ has sidelengths $AB = 10, AC = 14$, and, $BC = 16$. Circle ω_1 is tangent to rays $\overrightarrow{AB}, \overrightarrow{AC}$ and passes through B. Circle ω_2 is tangent to rays $\overrightarrow{AB}, \overrightarrow{AC}$ and passes through C. Let ω_1, ω_2 intersect at points X, Y. The square of the perimeter of triangle $\triangle AXY$ is equal to $\frac{a+b\sqrt{c}}{d}$, where a, b, c, and, d are positive integers such that a and d are relatively prime, and c is not divisible by the square of any prime. Find $a + b + c + d$.

7. Let $\triangle ABC$ be a triangle with $BC = 7$, $CA = 6$, and, $AB = 5$. Let I be the incenter of $\triangle ABC$. Let the incircle of $\triangle ABC$ touch sides BC, CA, and AB at points D, E, and F. Let the circumcircle of $\triangle AEF$ meet the circumcircle of $\triangle ABC$ for a second time at point $X \neq A$. Let P denote the intersection of XI and EF. If the product $XP \cdot IP$ can be written as $\frac{m}{n}$ for relatively prime positive integers m, n, find $m + n$.

8. Let $\triangle ABC$ have sidelengths $BC = 7$, $CA = 8$, and, $AB = 9$, and let Ω denote the circumcircle of $\triangle ABC$. Let circles $\omega_A, \omega_B, \omega_C$ be internally tangent to the minor arcs $\widehat{BC}, \widehat{CA}, \widehat{AB}$ of Ω, respectively, and tangent to the segments BC, CA, AB at points X, Y, and, Z, respectively. Suppose that $\frac{BX}{XC} = \frac{CY}{YA} = \frac{AZ}{ZB} = \frac{1}{2}$. Let t_{AB} be the length of the common external tangent of ω_A and ω_B, let t_{BC} be the length of the common external tangent of ω_B and ω_C, and let t_{CA} be the length of the common external tangent of ω_C and ω_A. If $t_{AB} + t_{BC} + t_{CA}$ can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n, find $m + n$.

(Write answers on next page.)

Name:

Team:

Write answers in table below:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8