P U M ∴ C

Number Theory A

1. Find the sum of all prime numbers p such that p divides

$$(p^2+p+20)^{p^2+p+2}+4(p^2+p+22)^{p^2-p+4}.$$

2. Compute the sum of all positive integers whose positive divisors sum to 186.

3. Given $k \geq 1$, let p_k denote the k-th smallest prime number. If N is the number of ordered 4-tuples (a,b,c,d) of positive integers satisfying $abcd = \prod_{k=1}^{2023} p_k$ with $a < b$ and $c < d$, find N (mod 1000).

4. Find the number of ordered pairs (x, y) of integers with $0 \leq x < 2023$ and $0 \leq y < 2023$ such that $y^3 \equiv x^2 \pmod{2023}$.

5. A positive integer $\ell \geq 2$ is called *sweet* if there exists a positive integer $n \geq 10$ such that when the leftmost nonzero decimal digit of n is deleted, the resulting number m satisfies $n = m\ell$. Let S denote the set of all sweet numbers ℓ. If the sum $\sum_{\ell \in S} \frac{1}{\ell - 1}$ can be written as $\frac{A}{B}$ for relatively prime positive integers A, B, find $A + B$.

6. Given a positive integer ℓ, define the sequence $\{a_n^{(\ell)}\}_{n=1}^{\infty}$ such that $a_n^{(\ell)} = \lfloor n + \sqrt[\ell]{n} + \frac{1}{2} \rfloor$ for all positive integers n. Let S denote the set of positive integers that appear in all three of the sequences $\{a_n^{(2)}\}_{n=1}^{\infty}$, $\{a_n^{(3)}\}_{n=1}^{\infty}$, and $\{a_n^{(4)}\}_{n=1}^{\infty}$. Find the sum of the elements of S that lie in the interval $[1, 100]$.

7. For a positive integer n, let $f(n)$ be the number of integers m satisfying $0 \leq m \leq n-1$ such that there exists an integer solution to the congruence $x^2 \equiv m \pmod{n}$. It is given that as k goes to ∞, the value of $f(225^k)/225^k$ converges to some rational number p/q, where p, q are relatively prime positive integers. Find $p + q$.

8. For $n \geq 2$, let $\omega(n)$ denote the number of distinct prime factors of n. We set $\omega(1) = 0$. Compute the absolute value of

$$\sum_{n=1}^{160} (-1)^{\omega(n)} \left\lfloor \frac{160}{n} \right\rfloor.$$

Name:

Team:

Write answers in table below:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8